ANALYSIS OF OPEN DIELECTRIC WAVEGUIDES USING MODE-MATCHING TECHNIQUE AND VARIATIONAL METHODS

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ABSTRACT

The mode-matching technique is employed for computing the propagation constants and field distributions of an inverted strip dielectric waveguide. The results derived in this manner are further improved by using variational formulas expressly designed for open dielectric waveguides. Illustrative numerical results are presented and compared with experimental measurements as well as those based on approximate methods found in the literature.

Introduction

Recent interest in the 30-300 GHz range, which has remained relatively unexplored hitherto, has led to the investigation of low-loss, low-cost dielectric waveguide designs[1-4] suitable for integrated circuit applications in this frequency range.

In order to develop reliable designs for uniform dielectric guides, as well as for active and passive components constructed from these waveguides, it is extremely important to have the capability of theoretically predicting the performance of these circuit elements and transmission media.

A search through the literature on optical and quasi-optical dielectric waveguides reveals, however, that the progress in this direction has been rather limited and the most commonly employed approach appears to be based on what is called the "effective dielectric constant" method[1-4]. An alternate approach called the "effective permeability" method has also been developed[5]; however, both of these techniques are based on certain approximations that are neither easily justified nor always satisfied. Furthermore, they do not provide complete field distributions. Recently, an exact formulation of the problem for dielectric image guides has been developed which is based on the expansion of the field in each subregion of the guide cross-section into a complete set of functions, and the consequent matching at the boundaries[6]. The numerical results obtained from this method seem to be in good agreement with the experimental results.

In this paper, a rather similar approach based on the mode-matching technique[7] is used for a more complete analysis of the open, planar, dielectric waveguide problem, specifically, homogeneous inverted strip guide (HIS). The method is quite general, and is useful even at optical frequencies. It appears that a one-mode approximation of the method presented here boils down to the "effective dielectric constant" approach.

The propagation constant obtained from the mode-matching technique is further improved by employing variational expressions which are modified for the present analysis.

Mode Matching

We consider the waveguide geometry in Fig. 1 which shows the cross-section of the homogeneous inverted strip guide (HIS). We investigate the problem of computing the propagation characteristics of both the fundamental and high-order modes in such waveguides. Numerical results are presented for the propagation constants as well as field distributions for various modes calculated at a frequency of 79.4 GHz. The method itself is quite general, and is useful even at optical frequencies. The major steps in the analysis are given:

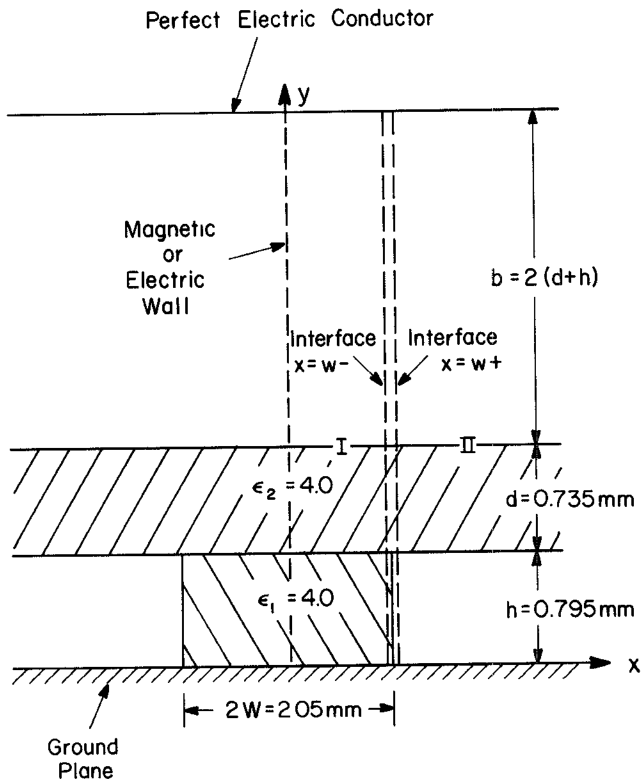

Figure 1. Cross-section of homogeneous inverted strip guide.

1. Invoke the symmetry of the configuration and insert a magnetic wall at x=0 for modes symmetric with respect to y. For antisymmetric modes, the magnetic wall is replaced by an electric wall.

2. Insert a top shield at y=h+d+b. Since, for guided modes, the fields are well-confined in both the x and y directions, we argue that these fields will be disturbed little if we insert a top shield sufficiently far above the guide. Insertion of the top shield transforms the original open-region problem into an equivalent closed-region problem, which is more easily manageable.

3. Express the field in the regions $0 \leq x \leq w^-$ and $w^+ \leq x$ in terms of a combination of TE and TM modes (with respect to the y-direction) whose coefficients are as yet unknown.

4. Impose the matching conditions for the transverse fields at the interface x=w.

5. Obtain the matrix eigenvalue equation using the moment method applied to the relationship derived in step 4.

6. Extract determinantal equation from the matrix eigenvalue equation. The roots of the determinantal equation **yield** the propagation constants.

7. Derive the corresponding modal field distributions for the different eigenvalues.

Before proceeding with the description of the numerical results, we mention an important theoretical result derived from this analysis that is useful in its own right. We have been able to demonstrate that the effective dielectric and permeability approaches fall out of our analysis if we restrict ourselves to a single-mode approximation. Thus, the present method, which is a generalization of existing approaches, enables us to derive higher-order approximations in a systematic manner.

Variational Improvement

The mode-matching results for the propagation constants can be further improved via the use of variational techniques. The field distribution is, in general, discontinuous across the interface of material discontinuity, due to the approximation introduced by the use of a finite number of modes in the process of matching the fields across the interface. The presence of such discontinuities requires that the conventional variational formulas for the propagation constant[8,9] be suitably modified. The modified formulas are then employed to obtain the propagation constants with improved accuracy.

Numerical Results

We present some representative results based on the analytical procedures already described. All of the results pertain to the homogeneous inverted strip guide of Figure 1, and are computed for 79.4 GHz to coincide with the experimental measurements.

Table I shows the convergence of the results for the propagation constant with the increase in the

TABLE I

PROPAGATION CONSTANT k_z (mm^{-1}) OF THE GUIDED MODES IN HOMOGENEOUS INVERTED STRIP GUIDE (FIG. 1) AT FREQUENCY 79.4 GHz

Mode	(Number of Terms) 1 TE 1 TM	3 TE 3 TM	5 TE 5 TM	7 TE 7 TM	Effective ε	Experiment
E^y_{11}	2.9718	2.9892	2.9872	2.9873	2.9906	3.0
H^y_{11}	2.7341	2.6210	2.7295	2.7203	2.7595	-
E^y_{21}	2.3646	2.3871	2.3910	2.3908	2.4070	

number of TE^y and TM^y modes retained in the mode-matching calculations. The results obtained from the "effective dielectric constant approach," which is equivalent to a single-mode approximation, and the measured values of k_z are also included in the table.

It should be mentioned that, unlike the "effective dielectric constant" approach, the mode-matching method produces accurate results for the field distribution of the dominant mode as well as the field distributions and propagation constants for higher-order modes.

Figures 2-5 show the comparison of the tangential field components of the E^y_{11} guide mode at $x=w^-$ and $x=w^+$. The H_y and H_z fields are extremely well-matched wnen 7 TE and 7 TM modes are used for the field expansion in the two regions, $0 \le x \le w^-$ and $w^+ \le x$. However, the E field match improves only slightly as the

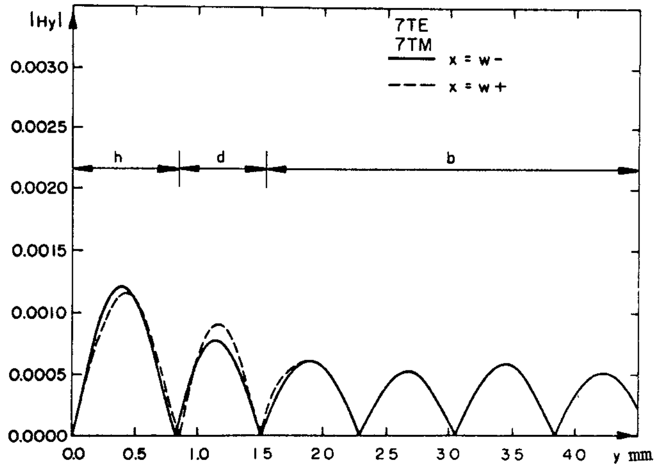

Figure 2. Plot of $|H_y|$ field of the E^y_{11} mode at the interface for $x=w^-$ and $x=w^+$.

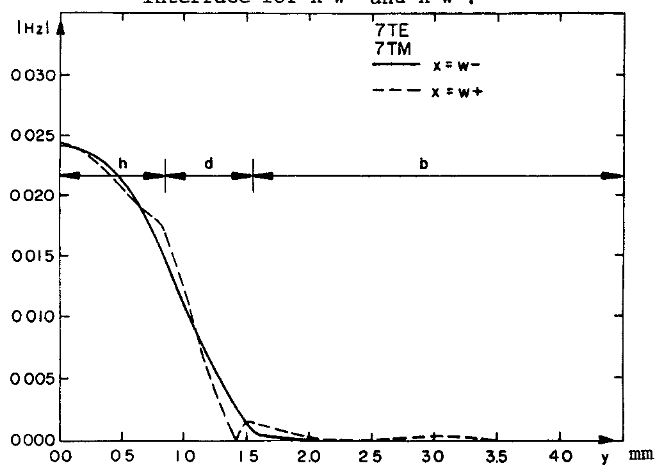

Figure 3. Plot of $|H_z|$ field of the E^y_{11} mode at the interface for $x=w^-$ and $x=w^+$.

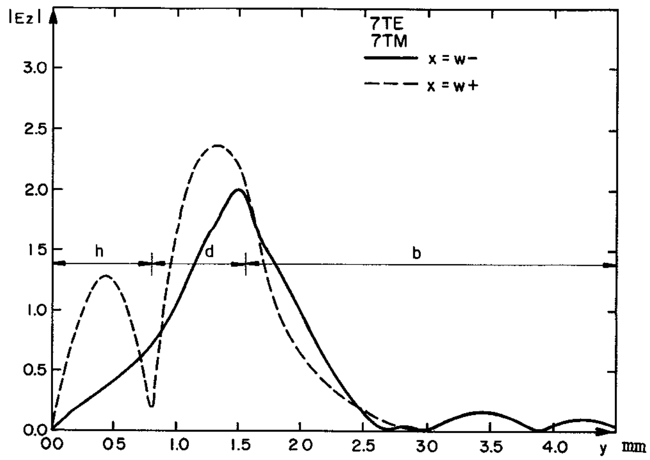

Figure 4. Plot of $|E_z|$ field of the E^y_{11} mode at the interface $x=w^-$ and $x=w^+$.

number of modes used is increased. The matching of E_y is more difficult since it is a continuous function of y at $x=w^-$ and $y=h$, but a discontinuous function of y at $x=w^+$ and $y=h$. However, except for this difficulty at $x=w$, matching process for the E_y field converges rapidly at other places as evident from Figures 6 and 7. Figure 6 shows the distribution of D_y at the $x=0$ plane. The field is extremely small at the top shield located at $y=h+d+b$, thus justifying our original assumption that the perturbation introduced by the perfect electric conductor placed at the top of the guide is negligible. Figure 7 shows the distribution of E_y at the $y=h+d+0^+$ plane. It is evident from this diagram that most of the energy carried by a guided wave is confined within the strip region.

The field distributions calculated via the mode-matching method are used in the modified variational formulas for the calculation of propagation constants. Some representative results using mixed-field variational formulas are presented in Table II and compared with the results obtained directly from the mode-matching calculation.

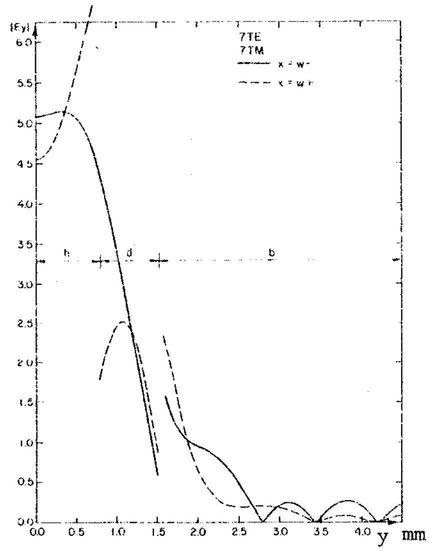

Figure 5. Plot of $|E_y|$ field of the E^y_{11} mode at the interface for $x=w^-$ & $x=w^+$.

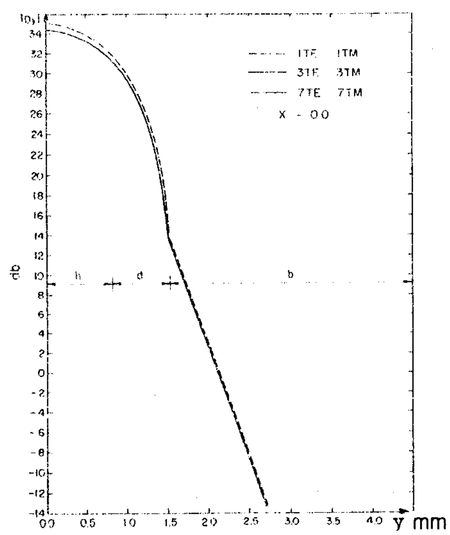

Figure 6. Plot of $|D_y|$ field of the E^y_{11} mode in vertical direction for $x=0.0$.

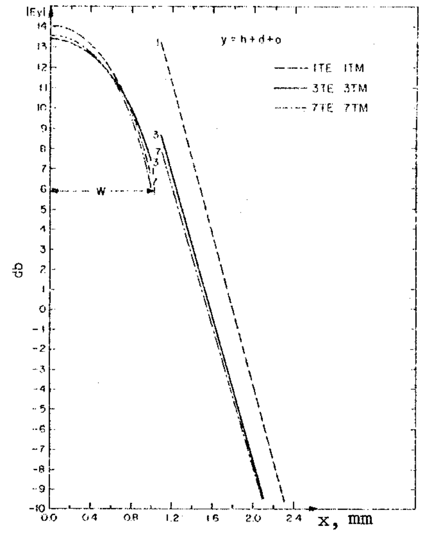

Figure 7. Plot of $|E_y|$ field of the E^y_{11}mode in transverse direction for $y=h+d+0$.

TABLE II. PROPAGATION CONSTANT $k_z (mm^{-1})$ CALCULATED VIA MIXED-FIELD VARIATIONAL FORMULA. COMPARISON IS MADE WITH MODE-MATCHING RESULTS (TABLE I).

		(Number of Terms)		
Mode	Method	1 TE 1 TM	3 TE 3 TM	5 TE 5 TM
E^y_{11}	Variational formula	2.9887	2.9859	
	Mode Matching	2.9718	2.9892	2.9872
H^y_{11}	Variational formula	2.7403	2.6138	--
	Mode Matching	2.7341	2.6210	2.7203
E^y_{21}	Variational formula	2.3949	2.3789	--
	Mode Matching	2.3646	2.3871	2.3910

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